ABSTRACTS

Tangent bundle and indicatrix bundle of a Finsler manifold

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Let \((M, F)\) be a Finsler manifold and \(G\) be the Sasaki-Finsler metric on \((TM, G)\). We show that the curvature tensor of the Levi-Civita connection on \((TM, G)\) is completely determined by the curvature tensor field of the Vranceanu connection and some well known Finsler tensor fields. Then we prove that \((TM, G)\) is locally symmetric if and only if \((M, F)\) is locally Euclidean. Next, for any non-zero \(c\) we define the \(c\)-indicatrix bundle \(IM(c)\) and obtain new and simple characterizations of Finsler manifolds of constant curvature \(c\). Finally we prove that if the scalar curvature of \((TM, G)\) is \(p\)-homogeneous of degree zero with respect to the fiber coordinates on \(TM\), and \(M\) is a compact connected and boundaryless manifold, then \((M, F)\) must be locally Minkowskian.

Generalized Lagrange metrics induced by Lorentz and Minkowski geometries

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By considering the norms provided by Lorentz or Minkowski (pseudo) inner product as influence functions, two oscillant distances can be generated in some subsets of Lorentz or Minkowski plane. In our main theorem, we show
that the infinitesimal arclength associated to those distances induces either Lorentz or Minkowski-type metrics. A geometric approach in constructing these metrics concludes that they are generalized Lagrange metrics. The theory is complemented by two examples.

Near-rings appeared in connection with geometry

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We study the history of near-rings, establishing that the near-rings appeared when the coordinatizations of some geometries have been considered. On the other side, the near-rings offer a lot of interesting examples in the theory of discrete geometries.

The geometry inspired some topics in the theory of near-rings as: planar near-rings, affine and generalized affine near-rings, applications of near-rings in combinatorics.

We recall some results of Hans Zassenhaus, Giovanni Ferrero, Gunter Pilz, Radu Miron and the author on this topic, completing them by new structure theorems.

References:


Parallel Morphisms in CR-submanifolds in a Locally Conformal Kaehler Manifold

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In this report, we consider several morphisms of distributions in a CR-submanifold in a locally conformal Kaehler (an l.c.K.-) manifold and we define covariant differentiations of these morphisms.

Next, we define the notion of parallel morphisms with respect to these covariant differentiations. Then, we mainly consider necessary and sufficient conditions that these morphisms are parallel. Finally, we consider such morphisms in a pseudo-umbilical CR-submanifold in an l.c.K.-manifold.

References:


Revisiting the Foundations of Barbilian’s Metrization Procedure: Domains with Two Associated Metrics

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Originally, in 1934, Barbilian’s metrization procedure induced a distance on a planar domain by a metric formula given by the so-called logarithmic oscillation. In 1959, Barbilian generalized this process to domains of a more general form, withstanding not necessarily on planar sets, but in a more abstract setting. Recently, it has been shown that Barbilian’s metrization procedure can be used to generate Riemannian and Lagrange generalized metrics. Furthermore, it has been proved that these Lagrange generalized metrics are irreducible to Finsler or Lagrange metrics. In the present work we show that one of Barbilian’s theorems from 1960 regarding the metrization procedure in the plane admits a natural extension to the n-dimensional real space, and it actually yields not only one metric in a given domain, but two, depending upon the relative position of two Apollonian hyperspheres of interest. More precisely, we are assigning a signature to the radii of Apollonian spheres, which leads us to the simultaneous coexistence of two metrics induced in the plane by Barbilian’s metrization procedure. Pursuing this viewpoint and revisiting the very foundations of Barbilian’s metrization procedure, we study the construction of these two metrics on some domains in the n-dimensional real space. Our examples are pointing out that Barbilian’s metrization procedure depends upon the points where the extrema are attained, and not the whole set whose influence induces the distance. Ultimately, the main point of our study is the understanding of conditions when it is possible to associate a generalized Lagrange metric and a Riemannian metric.
Atiyah’s conjecture revisited

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The Atiyah’s conjecture is a problem in elementary space geometry that has arisen from M.V.Berry and J.M. Robbins investigations on the spin-statistics theorem in quantum mechanics. Our aim is to state and prove a variation of Atiyah conjecture in the Euclidean plane which preserves the spirit of the space Atiyah’s conjecture.

On the existence of Landsberg structure

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After a short survey of some basic notions of Finsler structure and generalized Finsler structures on surfaces, we construct a specific linear Phaffian exterior differential system. Using it we prove a local existence theorem for generalized Landsberg structures on surfaces that are not Berwald type using the Cartan-Kahler theorem for linear Phaffian systems. Finally, we discuss the relation with classical Landsberg structures on surfaces. This is the joint work with Sabau Sorin.

Decomposable Hamilton spaces

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In the 2(n + m) dimensional Hamilton space of first order, $H$, some point $u \in H$ in some local chart has coordinates

$$u = (x^a, x^\alpha, p_a, p_\alpha), \quad a, b, c, \ldots = 1, n, \ alpha, \beta, \gamma, \ldots = 1, m.$$
The transformation group is given by \( C^\infty, 1 - 1 \) function of form:

\[
x^{\alpha'} = x^{\alpha'}(x^a), \quad p_{\alpha'} = B_{\alpha'}^\alpha p_\alpha,
\]

where

\[
B_a^{\alpha'} = \partial_a x^\alpha, \quad B_{\alpha'}^a = \partial_{\alpha'} x^a, \quad B_{\alpha'}^\alpha = \partial_{\alpha'} x^\alpha, \quad B_{\alpha}^{\alpha'} = \partial_{\alpha} x^{\alpha'}.
\]

Using the elements of

\[
\bar{B} = \left\{ \partial_a = \frac{\partial}{\partial x^a}, \partial_\alpha = \frac{\partial}{\partial x^\alpha}, \partial_{\alpha} = \frac{\partial}{\partial p_\alpha}, \partial_{\alpha'} = \frac{\partial}{\partial p_{\alpha'}} \right\},
\]

the natural basis of \( T(H) \) and the elements of

\[
\bar{B}^* = \{dx^a, dx^\alpha, dp_a, dp_\alpha\}
\]

the natural basis of \( T^*(H) \), the adapted bases \( B = \{\delta_a, \delta_\alpha, \delta^a, \delta^\alpha\} \) of \( T(H) \) and \( B^* = \{\delta x^a, \delta x^\alpha, \delta p_a, \delta p_\alpha\} \) of \( T^*(H) \) are determined in such a way that the elements of \( B \) and \( B^* \) are transforming as tensors and the elements of \( B^* \) are dual to elements of \( B \).

The general linear connection has \( 4^3 = 64 \) types of connection coefficients.

Different special kinds of linear connection, as almost \( d \)-connections, \( d \)-connection, strongly distinguished and almost strongly distinguished connections are defined.

The transformation law of connection coefficients are determined. Different covariant derivatives are obtained, which transform as tensors.

For mentioned different kind of covariant derivatives the torsion and curvature tensors are calculated.
General Connections on Lie Groupoids

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Using the Lie functor from the category of Lie groupoids into the category of Lie algebroids [constructed by Higgins and Mackenzie in [2]], we consider two concepts of connections defined on a Lie groupoid.

The main purpose of this paper is to establish some results for connections on Lie algebroids and Lie groupoids.

The study of these connections is important in the description of several geometrical objects in Lagrangian and Hamiltonian mechanics on Lie Groupoids and Lie algebroids, see for instance [4].

References:


Geodesic circles and the variational principle in two dimensions

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The unique variational formulation for geodesic circles in $\mathbb{R}^2$,

$$E_i = \frac{e_{ij} \ddot{u}^j}{\|u\|^3} - 3 \frac{(\dot{u} \cdot u)}{\|u\|^5} e_{ij} \ddot{u}^j + m \frac{\|u\|^2 \dddot{u}_i - (\dot{u} \cdot u) u_i}{\|u\|^3} = 0$$

is generalized to the (pseudo)Riemannian case. The covariant momenta $\pi_i$ and $\pi_i^{(1)}$ in the spirit of the higher-order Ostrohradskyj mechanics are introduced. The resulting third order Euler–Poisson equation $\mathcal{E}_i^R = 0$ involves the term $-\frac{1}{2} R_{il,kj} u^i \mathcal{G}^{kj}$ with the “spin” tensor $\mathcal{G}_{ij} = u_i \pi^{(1)}_j - u_j \pi^{(1)}_i$ that corresponds to the physical notion of spin-curvature interaction. This term constitutes the only byproduct of the formal replacement of ordinary derivatives in $\mathbb{R}^2$ by the covariant ones.

Study on the integrability of l.c.a.K.-structures and the relationships with almost contact structures

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An l.c.a.K.-manifold (locally conformal almost Kähler manifold) is an almost Hermitian manifold whose metric is conformal to an almost Kähler metric locally, and which is characterized by the existence of a closed 1-form $\alpha$ satisfying

$$d\Omega = 2\alpha \wedge \Omega$$

with the fundamental 2-form $\Omega$. (I.Vaisman)

A product manifold of a contact metric manifold and a unit circle is an example of an l.c.a.K.-manifold. Hence, we can obtain informations on contact metric manifolds from l.c.a.K.-geometry.
In this talk, we discuss mainly on integrability conditions of l.c.a.K.-structures making use of the Riemannian curvature tensor, and reconsider well known properties of contact metric manifolds from the relationship with l.c.a.K.-geometry.

A commuting property for generalized Hodge-de Rham operators

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Let \((M, g)\) be an \(n\)-dimensional, oriented, smooth Riemannian manifold, \(\mathcal{C}^\infty(M)\) the real algebra of smooth real functions on \(M\), \(A^k(M)\) the \(\mathcal{C}^\infty(M)\)-module of smooth differential \(k\)-forms, \(0 \leq k \leq n\), and \(\mathbf{h}\) a pointwise non-singular smooth tensor field of type \((1,1)\) on \(M\) with vanishing Nijenhuis derivation associated to \(\mathbf{h}\). For such \(\mathbf{h}\) there exists an associated exterior derivative \(d^{(k)}_h : A^k(M) \rightarrow A^{k+1}(M)\) and an associated codifferential \(\delta^{(k+1)}_h : A^{k+1}(M) \rightarrow A^k(M)\), so that one can define a (strongly) elliptic self-adjoint second order differential operator \(\Delta^{(k)}_h : A^k(M) \rightarrow A^k(M)\), which is a generalization of the usual Hodge-de Rham operator \(\Delta^{(k)} : A^k(M) \rightarrow A^k(M)\) (see [3]). We shall discuss some commuting properties for these \(\mathbf{h}\)-dependent differential operators (see also [6]).

References:


Some remarks on the calculus of variations

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We discuss a geometrical approach on the variational problem for 1-st and 2-nd order Lagrangians. Some applications and examples will be given in order to motivate this formalism. The whole theory is strictly related with classical Finsler geometry and Higher Order Geometries introduced by Prof. Radu Miron.

General harmonic theory

Mircea Craioveanu, Camelia-Ionela Petrișor
The West University of Timișoara, ROMANIA


In particular, another proof of the Hodge decomposition theorem is given.
Generalised Lagrange Spaces and almost symplectic conjugations

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In this paper the authors, starting from the general theory of the almost symplectic conjugations, which was elaborated by P. Stavre and from its applications to a Generalised Lagrange Space in R. Miron’s way, obtained by A. Lupu in his Ph.D. Thesis, will obtain the associated specific curvature invariants.

The Euler - Arnold equations in hamiltonian form of the heavy top

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The main purpose of this paper is to describe the dynamics of the heavy top in terms of the Hamiltonian formalism on a convenient Lie algebroid.

The paper contains three sections. In Section 2 we collect the definitions and basic results concerning the symplectic Lie algebroids. The third section is dedicated to description of the Hamiltonian formalism on the prolongation of the Lie algebroid associated to trivial vector bundle $E = M \times \mathfrak{g} \rightarrow M$ over its dual vector bundle $E^* = M \times \mathfrak{g}^* \rightarrow M$, where $\mathfrak{g}$ is a Lie algebra. Using the Hamiltonian formalism on a particularly transformation Lie algebroid, in Section 4 we present the dynamics in Hamiltonian form of heavy top. Finally, the Euler - Arnold equations in Hamiltonian form of the heavy top are determined.

References:


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**The almost metriplectic realization of the 4-dimensional periodic Volterra lattice**

**Mihai Ivan, Gheorghe Ivan**

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In this paper we refer to the almost metriplectic systems. A almost metriplectic structure on a smooth manifold is a pair of skew-symmetric Poisson tensor $P$ and symmetric tensor $g$. The dynamical system defined
by the almost metriplectic structure can be expressed in terms of left Leibniz bracket.

The periodic Volterra lattice is described by the following system of differential equations

\[ \dot{x}_i = x_i(x_{i+1} - x_{i-1}), \quad i = 1, 2, ..., n \]

(1)

where \( x_{n+1} = x_1 \) and \( x_0 = x_n \).

The aim of this paper is to present an almost metriplectic realization of the periodic Volterra lattice in the case when \( n = 4 \). For the revised dynamical system associated to the 4-dimensional periodic Volterra lattice we give the numerical integration via Kahan’s integrator.

Some remarks on controllability and integrability on the Lie group \( \text{SE}(2, \mathbb{R}) \times \text{SO}(2, \mathbb{R}) \)

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An optimal control problem on the Lie group \( \text{SE}(2, \mathbb{R}) \times \text{SO}(2, \mathbb{R}) \) is discussed and some of its geometrical and dynamical properties are pointed out.
Some properties of metallic structures defined on manifolds

Cristina-Elena Hrețcanu (Ștefan cel Mare University, Suceava, ROMANIA),
Mircea Claudiu Crăsmareanu (Al.I.Cuza University, Iași, ROMANIA),
Ciprian Ionel Hrețcanu (Ciprian Porumbescu College, Suceava, ROMANIA)

The purpose of this paper is to point out the close relations of particularly polynomial structures on Riemannian manifolds (named by us metallic Riemannian structures) and the almost product structures. The name of metallic structures is inspired by the Metallic Means Family which was introduced by V. Spinadel as a generalization of the Golden Mean. All the members of this family are solutions of quadratic equations of the type $x^2 - px - q = 0$ ($p,q \in \mathbb{N}^*$) for particularly values of $p$ and $q$. We say that an $(1,1)$-tensor field $J$ on a manifold $M$ is a metallic structure if it satisfies the equation $J^2 = p \cdot J + q \cdot I$ where $I$ is the identity $(1,1)$ tensor field. A Riemannian manifold $(M,g)$, endowed with a metallic structure $J$ so that the Riemannian metric $g$ is $J$-compatible (i.e. $g(JX,Y) = g(X,JY)$ for every tangent vector fields $X,Y \in \chi(M)$) is named a metallic Riemannian manifold and $(g,J)$ is named a metallic Riemannian structure on $M$.

A mathematical model for Wireless LAN indoor positioning system

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One of the most significant elements of context-awareness in ubiquitous environments is mobile device localization. To obtain the accurate location information for indoor environments is a challenging problem. There have been a number of attempts to design systems for indoor localization using different wireless sensing techniques. Signal strength (SS) approaches are
easy to implement in the existing technologies in mobile devices. In this field there are two main categories of signal strength (SS) based techniques for positioning using WLAN: trilateration and location fingerprint. We want to propose a method based on some kind of trilateration scheme with a simply and clearly mathematical model. The experimental results, show that the proposed solution is quite robust and gives accurate localization results.

**On the Geometry of Generalized Metric Spaces**

**Hiroaki Kawaguchi**  
 Tensor Society, JAPAN

The purpose of my talk is to construct a research concept of the geometry of generalized metric spaces, which consists of the following elements:

a) my research background,

b) generalization of Riemann metrics,

c) research aim of generalized metric spaces,

d) geometry of the higher order spaces.

This concept will suggest that we should continue our research on the generalized metric spaces with less confusion of the research aims.

**Geometrical aspects concerning the invariant of the general dynamics**

**Tomoaki Kawaguchi**  
 University of Tsukuba, JAPAN

It is not always geometrically invariant the general dynamics, there are thus examples not a few. Then the author introduces such things and suggests one of direction on the approach to the dynamical systems appearing frequently as the applications of geometry.
Dynamical Systems Associated to the Classical Mechanical Systems

Radu Miron
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One studies the canonical semispray associated to a non-conservative Riemannian mechanical system. The geometry of the pair $(TM, S)$ is the Lagrange geometry of the given non-conservative mechanical system.

Paraquaternionic manifolds and mixed 3–structures

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The paraquaternionic structures, firstly named quaternionic structures of second kind, have been introduced in geometry by P. Libermann, in 1952 [C.R. Acad. Sc. Paris, 234 (1952)].

The theory of paraquaternionic manifolds parallels the theory of quaternionic manifolds, but uses the algebra of paraquaternionic numbers, in which two generators have square 1 and one generator has square $-1$. Accordingly, such manifolds are equipped with a subbundle of rank 3 in the bundle of the endomorphisms, locally spanned by two almost product structures and one almost complex structure. From the metric point of view, the almost paraquaternionic Hermitian manifolds have neutral signature.

The counterpart in odd dimension of paraquaternionic geometry was introduced in by the present authors and R. Mazzocco [Mediterranean J. Math., 3 (2006), 3-4]. It is called mixed 3–structure, which appears in a natural way on lightlike hypersurfaces in paraquaternionic manifolds. We give examples of manifolds endowed with mixed 3–structures and obtain some properties. Particularly, we obtain that a compatible metric with a mixed 3–structures is necessarily semi-Riemann and mixed 3–Sasakian manifolds are Einstein, hence the possible importance of these structures in theoretical physics.
Bott type connections and vector valued cohomology on complex Finsler manifolds

Cristian Ida
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In this paper we define complex adapted connections of type Bott on vertical and horizontal bundles of a complex Finsler manifolds. When these connections are flat we get a vector valued cohomology. Also a cohomological criterion for integrability of horizontal distribution is given. The notions are introduced by analogy with real case for foliations.

Prolongation on Lie algebroids and its applications

Liviu Popescu
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In this paper we study the geometrical structures on the prolongation of Lie algebroids and the results are applied to Hamiltonian dynamics. The notion of Ehresmann connection associated to a regular section is naturally obtained. The notions of mechanical structures and semi-Hamiltonian sections are presented and their properties are investigated. Finally, some applications to optimal control are given.
Before Tzitzeica Surfaces

Wladimir G. Boskoff, Alexandru Bobe
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Some important problems of differential geometry were solved due to the famous Erlangen program of F. Klein who offered the idea to study geometry through certain groups of transformations. Following F. Klein's ideas, Gh. Tzitzeica has studied curves and surfaces and he obtained important affine, centroaffine and projective properties of the objects under the transformations and he discovered in 1907 a class of surfaces in the 3-dimensional space (S-surfaces) which are actually examples of today affine spheres. From the historical point of view, Gh. Tzitzeica was the first geometer who studied affine spheres using Euclidian invariants. In this paper, we’ll search for relations between the characteristic quantities of curves and surfaces which are preserved under the centroaffine transformations and this idea will lead us to the notion of Tzitzeica surfaces and to a possible answer to the question: "How and why Tzitzeica surfaces appeared?"

Remarks on the reduction of the standard $k$-symplectic and Poisson manifolds

Adara M. Blaga
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A canonical connection on a $k$-symplectic manifold is defined and sufficient conditions such that it should be preserved by performing Marsden-Weinstein reduction are given. In particular, the relation between the induced canonical connections on the reduced standard $k$-symplectic manifolds with respect to the action of a Lie group $G$ is established. Similarly, defining Poisson structures on these manifolds, the relation between the corresponding Poisson structures on the reduced manifolds is stated.