# ABSTRACTS

**International Conference & Humboldt Kolleg "Fundamental Structures of Algebra"**
*In honor of the 70th birthday of Professor SERBAN BASARAB*
***Organizers**: the Faculty of Mathematics and Computer Science, Ovidius University Constanta, the Mathematical Institute "Simion Stoilow" of the Romanian Academy Bucharest, the Romanian Mathematical Society in association with*

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## *From field theoretic to abstract Cogalois Theory*

**Toma ALBU**
Institute of Mathematics "Simion Stoilow" of the Romanian Academy, Bucharest, ROMANIA

The aim of this talk is to present to a general audience some of the basic concepts, results, and applications of Cogalois Theory, a fairly new area in Field Theory investigating field extensions, finite or not, that possess a Cogalois correspondence. The subject is somewhat dual to the very classical Galois Theory dealing with field extensions possessing a Galois correspondence. As there exists an Abstract Galois Theory for arbitrary profinite groups, an Abstract Cogalois Theory for such groups has been recently invented and outlined in this talk.

*Syzygies of curves on K3 surfaces (joint work with G. Farkas)*

**Marian APRODU**
Institute of Mathematics "Simion Stoilow" of the Romanian Academy, Bucharest, ROMANIA

Green's conjecture predicts that the shape of the Betti tables of canonical curves are completely determined by the Clifford indices. We present a proof of Green's conjecture for any smooth curve on an arbitrary K3 surface. This result has a particular interest, due to Green's hyperplane section theorem. Our result implies that the shapes of Betti tables of projective K3 surfaces are determined by the Clifford indices of corresponding hyperplane sections.

*The space of real places of real function fields*

**Eberhard BECKER**
Dortmund University, GERMANY

If $F$ is a real function field over the field of real numbers it is well known that its space of real places is the projective limit of the spaces of real points of all its smooth projective models. This relationship will be studied from a topological point of view and via the interplay with various holomorphy rings in the function field. It is a final objective to understand previous results of Schülting, Ischebeck and Kucharcz in a more comprehensive framework, not yet established.

*Coset-minimal ordered groups*

**Oleg BELEGRADEK**
Istanbul Bilgi University, TURKEY

A linearly ordered group (possibly, with extra structure) is called coset-minimal if every definable subset of it is a finite union of cosets of definable subgroups intersected with intervals. The simplest example of a coset-minimal group is the ordered group of integers. We will give a survey of results on coset-minimality and related notions.

## *Reciprocity laws for Legendre symbols of the type $(a + b\sqrt{m}|p)$*

**Constantin-Nicolae BELI**
Institute of Mathematics "Simion Stoilow" of the Romanian Academy, Bucharest, ROMANIA

We define two functions involving Legendre symbols of the type $(a + b\sqrt{m}|p)$, in particular the rational quartic residue symbols $(a|p)_4$, and we state their properties. As an application of these properties we obtain some very general reciprocity laws.

We show how our result can be used to prove many quartic reciprocity theorems previously obtained by Gauss, Dirichlet, E. Lehmer, Burde, Scholz and others.

## *From prime numbers to irreducible multivariate polynomials*

**Nicolae BONCIOCAT**
Institute of Mathematics "Simion Stoilow" of the Romanian Academy, Bucharest, ROMANIA

We present some recent irreducibility results for multivariate polynomials, that include methods to test the irreducibility of multivariate polynomials over arbitrary fields, as well as methods to produce irreducible polynomials starting from irreducible polynomials in fewer variables, in particular from prime numbers.

## *Twisted Fourier-Mukai transform and applications*

**Vasile BRINZANESCU**
Institute of Mathematics "Simion Stoilow" of the Romanian Academy, Bucharest, ROMANIA

We shall present some facts on twisted sheaves and the definition of the twisted Fourier-Mukai transform. Some applications to moduli spaces of stable vector bundles will be given.

## *On Brauer groups, valuations and decomposition groups of quasilocal fields*

**Ivan D. CHIPCHAKOV**
Institute of Mathematics and Informatics, Bulgarian Academy of Sciences

Abstract. This talk describes, up-to an isomorphism, the abelian torsion groups realizable as Brauer groups of Henselian valued primarily quasilocal fields with totally indivisible value groups. When $E$ is a quasilocal field of this kind, it shows that the Brauer group $\mathrm{Br}(E)$ is divisible and embeddable in the group of complex roots of unity. At the same time, it is obtained that every divisible abelian torsion group $T$ is isomorphic to $\mathrm{Br}(F)$, for some quasilocal field $F$ with projective decomposition Galois groups, for each Krull valuation of $F$. When the p-components of T are nontrivial, this allows us to construct $F$ so that its proper finite Galois extensions violate the Hasse norm principle.

## *Small solutions to systems of polynomial equations with integer coefficients*

**Mihai CIPU**
Institute of Mathematics "Simion Stoilow" of the Romanian Academy, Bucharest, ROMANIA

This talk contains a discussion of a series of conjectures due to A. Tyszka aiming to describe boxes in which there exists at least one solution to a system of polynomial equations with integer coefficients. A proof of the bound valid in the linear case is given.

## *Model theory of difference fields and applications*

**Zoe CHATZIDAKIS**
CNRS - Paris 7, FRANCE

A difference field is a field with a distinguished automorphism. The model theory of the existentially closed difference fields has been extensively studied, and I will recall some of the important results. I will particularly insist on the description of one-based sets, recall their properties, and how they can be used towards applications. At the end of the talk, I will mention a recent application to a problem of Szpiro. (Joint work with E. Hrushovski).

## *Fuzzy grade and intuitionistic fuzzy grade of hypergroups*

**Irina CRISTEA**
Udine University, ITALY

The aim of this talk is to present a connection between Fuzzy Sets and Algebraic Hyperstructures. More precisely, we will give a survey on the fuzzy grade of a hypergroup. With any hypergroup $H$ one can associate a sequence of join spaces and fuzzy sets, whose length is called fuzzy grade of $H$. We will recall some of the principal results obtained on this topic in the general case, and also for some particular classes of hypergroups (the complete hypergroups and the i.p.s. hypergroups). Finally we will describe some generalizations of the above mentioned sequence, starting from a fuzzy set endowed with two membership functions and then, from an intuitionistic fuzzy set.

Joint work with Mirela Ştefănescu, Ovidius University of Constanta.

## *Relative categoricity in abelian groups III*

**Wilfrid HODGES**
Queen Mary, University of London, UK

In 2009 Anatoly Yakovlev and I published a long paper in which we classified complete theories of pairs of abelian groups according to their spectra of relative categoricity. We ran out of space before we had covered all the model-theoretic implications. My talk will describe some of what we had to leave out. It includes questions about the relationship between algebraic and model-theoretic methods, a field to which Serban Basarab has contributed much insight.

## *Frobenius endomorphisms on matrix spaces*

**Alexander GUTERMAN**
Moscow State University

The theory of Frobenius endomorphisms, i.e., transformations preserving different properties and invariants dates back to the works by Frobenius, Schur, and Dieudonné, and is an intensively developing part of algebra nowadays.

The problem can be formulated as follows. Let $T : M_n(R) \to M_n(R)$ be a certain transformation on matrices of a fixed order $n$ over a certain ring $R$. Let us consider a subset $S \subseteq M_n(R)$, or a matrix functional $\rho : M_n(R) \to Q$, where $Q$ is a given set ($\rho$ can be a determinant, trace, rank, permanent, etc.), or a matrix property $\mathcal{P}$ (nilpotence, idempotence, singularity, etc.), or a matrix relation $\mathcal{R}$ (similarity, commutativity, order, etc.). It is assumed that one of the following holds: the transformation $T$ *preserves* the set $S$, or the functional $\rho$, or the property $\mathcal{P}$, or the relation $\mathcal{R}$, which means that $X \in S$ implies $T(X) \in S$; $\rho(X) = \rho(T(X))$ for all $X \in M_n(R)$; if $X$ satisfies $\mathcal{P}$, then $T(X)$ satisfies $\mathcal{P}$ also; and $X\mathcal{R}Y$ implies $T(X)\mathcal{R}T(Y)$, correspondingly.

Such maps are usually called *Frobenius endomorphisms*.

The main problem is to characterize all Frobenius endomorphisms preserving one of $S$, $\rho$, $\mathcal{P}$, or $\mathcal{R}$, possibly under some additional assumptions such as linearity, additivity, bijectivity, etc.

In this talk we give an overview of the development of this field including our recent results.

## *Valuations on semirings and supervaluations*

**Manfred KNEBUSCH**
Universitaet Regensburg, GERMANY

Bourbakis notion of a valuation on a ring (Algebre commutative VI) readily generalizes to semirings, but, of course, studying these leads to new - in fact rather interesting - phenomena. Such a valuation can best be viewed as a map into another semiring of special nature (a "bipotent" semiring). It is possible to lift this map to a map of special type with values in a so called "supertropical semiring" in various ways, which reveals a sort of complexity of the given valuation. These are the "supervaluations". They

seem to deserve attention on their own in connection with tropical geometry. Valuations on semirings take a natural place in real algebra. The semiring of all sums of squares in a field is a case in point.

Joint work with Zur Izhakian and Louis Rowen (Bar Ilan University)

## *Quantifier Elimination for Valued Fields*

**Franz-Viktor KUHLMANN**
University of Saskatchewan, Saskatoon, CANADA

I will give a survey on the known Quantifier Elimination results for valued fields, introducing various languages in which QE has been achieved. In particular, I will describe the approach of Serban Basarab (Basarab, Serban A.: Relative elimination of quantifiers for Henselian valued fields. Ann. Pure Appl. Logic 53 (1991), no. 1, 51-74) and a further refinement due to myself (Kuhlmann, Franz-Viktor: Quantifier elimination for Henselian fields relative to additive and multiplicative congruences. Israel J. Math. 85 (1994), no. 1-3, 277-306). Interconnected with this is the description of the finite extensions of a henselian field of residue characteristic 0 (Basarab, Serban A.; Kuhlmann, Franz-Viktor: An isomorphism theorem for Henselian algebraic extensions of valued fields. Manuscripta Math. 77 (1992), no. 2-3,113-126). Among others, I will sketch the approaches of Francoise Delon (in her thesis) and of Johan Pas. I will also describe the main open problems. For valued fields in positive characteristic which are not Kaplansky fields, we have QE only in very special cases. Although I was able to prove Ax-Kochen-Ershov principles for tame valued fields, QE is in general not known for them. The obstacle in this case is that we do not know a structure like (or in addition to) the additive and multiplicative congruences that could describe the algebraic extensions of a henselian field with positive residue characteristic.

## *On a minimal language for the semi-algebraic subsets of p-valued fields*

**Eva LEENKNEGT**
Katholieke University of Leuven, BELGIUM

Given some valued field $K$, one can consider the semi-algebraic sets contained in $K$ itself, i.e. the subsets of $K$ definable using the language of valued fields. It follows from cell decomposition results that the semi-algebraic subsets of $K$ can also be defined using weaker languages. For example, we can omit the symbol for the multiplication map, if we take care that the relation 'is an n-th power' remains definable. We can define a partial order relation on all languages in which the semi-algebraic subsets of K are definable. It is then natural to look for a minimal language, i.e. a minimal element of this order relation.

In this paper, we propose a candidate for such a minimal language for a large class of valued fields. Using cell decomposition techniques, we will show that this language has no non-trivial definable functions, so from that point of view our language is very weak. However, it is still rather strong, as the full p-adic topology is definable in our language.

## *Projective manifolds covered by lines*

**Paltin IONESCU**
University of Bucharest and Institute of Mathematics "Simion Stoilow" of the Romanian Academy, Bucharest, ROMANIA

We investigate recent aspects in the classification theory of embedded complex projective manifolds, say $X$, with the property that given any point $x$ in $X$, there exists some line contained in $X$ and containing $x$.

## *Model Theory of Adeles Revisited*

**Angus MACINTYRE**
Queen Mary, Westfield College, University of London, UK

## *Cuspidal locally complete intersection nilpotent structures*

**Nicolae MANOLACHE**
Institute of Mathematics "Simion Stoilow" of the Romanian Academy, Bucharest, ROMANIA

We give constructions for locally complete intersection nilpotent structures $Y$ on a smooth variety $X$ as support, embedded in a smoth variety $P$, which are locally (i.e. after taking the completions of the local rings) given by ideals of the form $(xy, y^2 + x^n, u_1, \ldots, u_m)$ in convenient local coordinates (in these coordinates the respective point on $P$ has the (completed) local ring of the form $k[[t_1, \ldots, t_c, x, y, u_1, \ldots, u_m]]$, and on $X$ the point has the local ring $k[[t_1, \ldots, t_c]]$). We give also a construction for lci complete intersection structures such that in all points the ideal has the form $(xy, y^3 + x^n, u_1, \ldots, u_m)$. These two constructions, together with the known constructions for quasiprimitive structures, give in particular the possibility to classify lci nilpotent structures of multiplicity 7 on a smooth support.

## *A class of groups universal for free $\mathbb{R}$-tree actions*

**Thomas W. MÜLLER**
Queen Mary, Westfield College, University of London, UK

I report on a new construction in group theory giving rise to a kind of continuous analogue of free groups. More explicitly, given any (discrete) group $G$, we construct a group $\mathcal{RF}(G)$ equipped with a natural (real-valued) Lyndon length function, and thus with a canonical action on an associated $\mathbb{R}$-tree $\mathbf{X}_G$, which turns out to be transitive. Analysis of these groups $\mathcal{RF}(G)$ is difficult. However, conjugacy of hyperbolic elements is understood, as are the centralizers and normalizers of hyperbolic elements; moreover, we show that $\mathcal{RF}$-groups and their associated $\mathbb{R}$-trees are *universal* (with respect to inclusion) for free $\mathbb{R}$-tree actions. Furthermore, we prove that

$$|\mathcal{RF}(G)| = |G|^{2^{\aleph_0}},$$

and that non-trivial normal subgroups of $\mathcal{RF}(G)$ contain a free subgroup of rank $|\mathcal{RF}(G)|$, as well as a number of further structural properties of $\mathcal{RF}(G)$ and its quotient by the span of the elliptic elements.

## *Untwisting cohomology*

**Ştefan PAPADIMA**
Institute of Mathematics "Simion Stoilow" of the Romanian Academy, Bucharest, ROMANIA

I will discuss several facets of twisted cohomology (group-theoretic, algebro-geometric, topological).

## *A matrix version of harmonic analysis*

**Nicolae POPA**
Institute of Mathematics "Simion Stoilow" of the Romanian Academy, Bucharest, ROMANIA

The aim of my talk is to give some parallel statements in both classical harmonic analysis and in matriceal harmonic analysis. In fact matriceal harmonic analysis isa theory of infinite matrices based on the similarity between the Fourier coefficients of a periodical function (or distribution) and diagonals of an infinite matrix.

So we give some results about Bergman-Schatten spaces which are the analogues of known theorems about Bergman spaces and also some characterization of classical Schatten spaces of matrices.

An important role in this talk is playing by Schur multipliers.

## *Bounds of Stanley depth*

**Dorin POPESCU**
Bucharest University, ROMANIA

Let $S = K[x_1, \ldots, x_n]$ be the polynomial $K$-algebra in $n$-variables over a field $K$ and $I \subset S$ a monomial ideal. Herzog, Takayama and Terai proved that *depth* $S/I \leq$ *depth* $S/\sqrt{I}$. Recently Ishaq showed in particular that the same inequality holds for the Stanley depth. Using this result we will report on different bounds of Stanley depth.

## *On places of algebraic function fields*

**Alexander PRESTEL**
Konstanz University, GERMANY

Let $F/K$ be an n-dimensional algebraic function field of characteristic 0. We shall consider the space $X$ of all (arbitratry) places on $F/K$ in a very fine topology and show that the set of iterated prime divisors lies dense in $X$. The proof uses the Ax-Kochem, Ershov Principle and Local Uniformisation. This result allows several applications.

## *Group Algebras and von Neumann algebras*

**Florin RADULESCU**
Institute of Mathematics "Simion Stoilow" of the Romanian Academy, Bucharest, ROMANIA

We present various aspects of von Neumann algebras that reveal the rich "Borel structure" of group algebras. Most interesting examples come from free non-belian groups.

## *What is reciprocity?*

**Peter ROQUETTE**
Math. Inst. Univ. Heidelberg, Heidelberg GERMANY

From quadratic reciprocity to Artin's general reciprocity law and Hasse's local global principle. A historical review, based on letters and documents found in the Goettingen archives and elsewhere.

## *Igusa's local zeta function and its poles*

**Dirk SEGERS**
Katholieke University of Leuven, BELGIUM

The numerical data of an embedded resolution determine the candidate poles of Igusa's $p$-adic zeta function. We determine which real candidate poles are actual poles in the curve case.

## *Zeta functions and jet spaces*

**Willem VEYS**
Katholieke University of Leuven, BELGIUM

We explore the relation between poles of Igusa, respectively motivic zeta functions and divisibility of numbers of solutions of associated congruences, respectively of associated jet spaces.