On the number of polynomials with coefficients in \([n]\)

Dorin Andrica and Eugen J. Ionascu

" Babes-Bolyai“ University, Faculty of Mathematics and Computer Science, Cluj Napoca, Romania and
Columbus State University, Math Department, Columbus, USA

Abstract

In this paper we introduce several sequences related to polynomials of degree \(s\) having coefficients in the set \(\{1,2,\ldots,n\}\), where \(n\) is a positive integer, which factor completely over the integers. These sequences can be seen as generalizations of A006218. We provide precise methods for calculating the terms and investigate the asymptotic behaviour of these sequences for \(s = 1, 2, 3\). Some open problems and directions of research concerning these sequences are also formulated.

Asymptotic evaluations for certain arithmetic functions

Magdalena Bănescu

Institute of Geodynamics of the Romanian Academy, Bucharest, Romania

Abstract

In this paper we give some asymptotic evaluations for some arithmetic functions. We use parabola method and a generalization to functions of several variables.

Combinatorics on Finite Fields

Şerban Bărcănescu

" Simion Stoilow“ Institute of Mathematics of the Romanian Academy, Bucharest

Abstract

Some combinatorial properties of the finite prime fields are presented, in connection
with the natural action of the multiplicative group on suitable symmetric modules, thus obtaining an interpretation of the equations verified over the rationals by the gaussian 2 and 3-periods. The constructions given are easy generalizable, modulo increasing computational effort. The main difficulty lies, as in some other major questions concerning the finite fields, in the understanding of the deep connection between the additive and the multiplicative structures of these fields.

A new type of quadratic forms in characteristic two

Constantin Nicolae Beli

"Simion Stoilow" Institute of Mathematics of the Romanian Academy, Bucharest

Abstract

Let $F$ be a field. We denote by $\text{Br}(F)$ the torsion of its Brauer group $\text{Br}(F)$. If $\text{char } F \neq 2$ then we have the Hilbert symbol

$$(\cdot , \cdot ) : F^*/F^{*2} \times F^*/F^{*2} \to \text{Br}(F),$$

where for any $a, b \in F^*$, $(a, b)$ is the image in $\text{Br}(F)$ of the quaternion algebra generated by 1, $i, j, ij$ subject to $i^2 = a, j^2 = b, ij + ji = 0$. If $\text{char } F = 2$ then, besides the group $(F^*/F^{*2}, \cdot )$, we have the groups $(F/F^{*2}, +)$ and $(F/\varphi(F), +)$, where $\varphi : F \to F$ is the Artin-Schreier operator $\varphi(x) = x^2 + x$. This time we have two symbols:

$$[\cdot , \cdot ] : F/\varphi(F) \times F^*/F^{*2} \to \text{Br}(F)$$

$$((\cdot , \cdot )) : F/F^{*2} \times F/F^{*2} \to \text{Br}(F)$$

If $a, b \in F^*$ then $[a, b] \in \text{Br}(F)$ is defined by the quaternion algebra generated by 1, $i, j, ij$ subject to $i^2 = a, j^2 = b, ij + ji = 1$. We have $[(a, b)] = [ab, b]$ if $b \neq 0$ and $[(a, b)] = 0$ if $b = 0$. If $\text{char } F \neq 2$ then a nondegenerate quadratic space $V$ is characterized by three invariants, $\dim V$, $\text{det } V \in F^*/F^{*2}$ and $S(V) \in \text{Br}(F)$, the Hasse invariant. If $F$ is a local field, $F \neq R$, these invariants fully determine the class of $V$.

If $\text{char } F = 2$ then we have again three invariants, $\dim V$, $\text{Arf } V \in F/\varphi(F)$ if $\dim V$ is even or $\text{det } V \in F^*/F^{*2}$ if $\dim V$ is odd and $C(V) \in \text{Br}(F)$, the Clifford invariant. If $F$ is a local field these invariants fully determine the class of $V$.

In a certain way the quadratic spaces in characteristic $\neq 2$ are related to the group $F^*/F^{*2}$ and the symbol $(\cdot , \cdot )$, while in characteristic 2 they are related to the groups $F/\varphi(F)$ and $F^*/F^{*2}$ and the symbol $[\cdot , \cdot ]$. E.g. assume that $V,W$ are nondegenerate quadratic spaces of dimensions $m$ and $n = m-1$. In characteristic $\neq 2$ if $W$ is represented by $V$ then $S(V) = S(W) + (\text{det } W, -\text{det } V)$, while in characteristic 2 the condition is that $C(V) = C(W) + [\text{Arf } V,$
detW) if m is even and it is that C(V ) = C(W) + [Arf W, det V ) if m is odd. When F is a local field other than R these conditions are also sufficient.

The new object that we introduce here, the new quadratic spaces, are characterized by 4 invariants: dim V , type V ∈ {0,1}, SumV ∈ F/F^2 and C(V ) ∈ 2 Br(F). If F is a local field these invariants fully determine the class of V . When dim V = dimW + 1 if V represents W then type V ≥ typeW and C(V ) = C(W) + ((SumV, SumW)). If F is a local field these conditions are also sufficient.

Hence we may say that the new quadratic forms are related to the group F/F^2 and the symbol (⟨·, · ⟩).

About k-perfect numbers

Mihály Bencze

“Áprily Lajos“ College, Brasov, Romania

Abstract

In 1974 M. Bencze introduced the notion of k-perfect number, defined in the following way: σ(n) = kn, where k is a positive integer. In this paper we present some interesting inequalities for the k-perfect numbers.

Effective results for Diophantine Equations over finitely generated domains

Attila Bérczes

University of Debrecen, Hungary

Abstract

Let A be an arbitrary integral domain of characteristic 0 that is finitely generated over Z. We consider Thue equations F(x, y) = δ in x, y ∈ A, where F is a binary form with coefficients from A and δ is a non-zero element from A, and hyper- and superelliptic equations f(x) = δ y^m in x, y ∈ A, where f ∈ A[X], δ ∈ A \ {0} and m ∈ Z_{≥2}.

Under the necessary finiteness conditions we give effective upper bounds for the sizes of the solutions of the equations in terms of appropriate representations for A, δ, F, f, m. These results imply that the solutions of these equations can be determined in principle. Further, we consider the Schinzel-Tijdeman equation f(x) = δ y^m where x, y ∈ A and m ∈ Z_{≥2} are the unknowns and give an effective upper bound for m.

In the proofs we combine effective finiteness results for these types of equations over number fields and over function fields, along with a specialization method developed by Győry in the 1980's and refined recently by Evertse and Győry.
This is joint work with J.-H. Evertse and K. Győry.

Prime numbers and bicrossproducts of finite groups

Nicolae Ciprian Bonciocat

” Simion Stoilow“ Institute of Mathematics of the Romanian Academy, Bucharest

Abstract

We present Takeuchi’s bicrossproduct construction and the corresponding notion of a matched pair of groups, and give some of their basic properties. We then give necessary and sufficient conditions for a pair of groups to be matched by automorphisms, and show that a pair of finite cyclic groups can be matched by automorphisms, provided some exponential congruences are satisfied. Then, by studying the solutions of these congruences in the symmetric case when the two groups coincide, as well as their mutual group actions, we obtain some curious primality conditions. Prime numbers appearing in this way are then investigated.

Fuzzy subhypergroup degree

Irina Cristea

Centre for Systems and Information Technologies, University of Nova Gorica, Slovenia

Abstract

The first connection between algebraic structures and fuzzy sets was established by A.Rosenfeld in 1971, when he introduced the concept of fuzzy group and gave the characterization of fuzzy subgroups. Based on similarities between groups and hypergroups, in 1999 B. Davvaz extended the notion of fuzzy subgroup to the general case of fuzzy subhypergroup. In this presentation, we define the degree to which a fuzzy subset is a fuzzy subhypergroup of a given hypergroup by means of residuate sets. Furthermore, the main properties are investigated in the particular case of polygroups.
Some interesting prime numbers

Alexandru Gica

University of Bucharest

Abstract

The aim of our research is to find all primes $p$ which have the property that $p+x^2$ has at most two prime divisors for all odd integers $x$ such that $x^2<p$. We are able to solve completely this problem in the cases $p=8k+1$, $p=8k+3$ and $p=8k+5$. The proof in these cases relies upon a computation of a class number of a quadratic imaginary field. In the case $p=8k+7$ we were able to show that there are only a finitely number of solutions.

Primes represented by ternary cubic forms

Jordi Guàrdia

University Politècnica de Catalunya, Spain

Abstract

The classical arithmetic problem of representing a prime number as a sum of two squares has originated a number of important advances in Number Theory, from the composition of binary quadratic forms to class field theory. The representation of primes by ternary cubic forms was also considered in the 18th century, but its technical difficulties and the development of modern class field theory led to a more general setting of the problem. The work of Bhargava and the present computational facilities constitute a new framework to consider again ternary cubic forms and their arithmetic applications. In the talk we plan to explain what is known about the representation of primes by ternary cubic forms and present our results in some particular cases.

This is joint work with Diana Savin and Montse Vela.

On the sup-norm problem for arithmetic hyperbolic 3-manifolds

Gergely Harcos

"Alfréd Rényi" Institute of Mathematics, Hungarian Academy of Sciences, Budapest

Abstract

I will talk about the sup-norm of cusp forms on arithmetic hyperbolic 3-manifolds. This is joint work with Valentin Blomer and Djordje Milicevic.
Non–square order Tate–Shafarevich groups of abelian surfaces over the rationals

Stefan Keil
Humboldt University of Berlin, Germany

Abstract

For an elliptic curve (over a number field) it is known that the order of its Tate-Shafarevich group is a square, provided it is finite. In higher dimensions this no longer holds true. We will present work in progress on the classification of all occurring non-square parts of orders of Tate-Shafarevich groups of non-simple abelian surfaces over the rationals. We will prove that only finitely many cases can occur. To be precise only the cardinalities k=1,2,3,5,6,7,10,13,14,26 are possible. So far, for all but the last three cases we are able to show that these cases actually do occur by constructing explicit examples.

Finite geometric spaces, Steiner systems and cooperative games

Antonio Maturo and Fabrizio Maturo
University of Chieti, Italy

Abstract

Some relations between finite projective or affine spaces and cooperative games are considered. Moreover the concept of projective or affine space on a Galois field GF(q), where q is a power of a prime, is generalized with the one of “generalized projective or affine space” on a finite ring R. Their properties depend on the numeric properties of the order n of R. Some blocks of these spaces and their dependence from n are investigated. Finally, possible applications to cooperative games are studied.

Unramified Iwasawa module of $\mathbb{Z}_p$-extensions

Ali Mouhib
University of Fes, Faculty of Taza
LIMAO, Department of Mathematics, Physics and Computer Science

Abstract

We give a survey of interesting results on Iwasawa theory of $\mathbb{Z}_p$-extensions of number fields. For a number field k and a prime number p, let $k_\infty$ be the cyclotomic $\mathbb{Z}_p$-extension of
k. We study the finiteness of the Galois group $\mathcal{X}_\infty$ over $k_\infty$ of the maximal abelian unramified $\mathbb{Z}_p$-extension of $k_\infty$. We then focus our attention to the case where $k$ is a cyclic extension of degree $p$.

**False complex multiplication on elliptic curves**

Filip Najman

Department of Mathematics, University of Zagreb, Croatia

Abstract

We introduce the notion of false complex multiplication on elliptic curves, and describe certain classes of so called $\mathbb{Q}$-curves with this property. As one of the consequences, we prove that any elliptic curves over any quadratic field with a point of order 13 or 18 and any elliptic curve over any quartic field with a point of order 22 has even rank.

**Higher moments for quadratic Dirichlet L-functions II**

Vicenţiu Paşol

”Simion Stoilow“ Institute of Mathematics of the Romanian Academy, Bucharest

Abstract

We construct the "improved" Multiple Dirichlet Series for quadratic Dirichlet L-functions for all moments over any number field.

This is joint work with Adrian Diaconu (U. Minnesota).

**On generalized Lebesgue-Ramanujan-Nagell equations**

István Pink

University of Debrecen, Hungary

Abstract

Let $d$ be a non-zero integer and $f(x)$ be a quadratic polynomial with integer coefficients. Further, we assume that the prime factors of the (non-zero) discriminant of $f(x)$ belong to a given finite set.
In our talk we give a brief overview of the results and methods related to the Diophantine equation \( f(x) = dy^n \), where \( x > 1, y > 1 \) and \( n > 3 \) are unknown integers.

A simple proof of the Eichler-Selberg trace formula

Alexandru Popa

”Simion Stoilow“ Institute of Mathematics of the Romanian Academy, Bucharest

Abstract

I will give a simple algebraic proof of the Eichler-Selberg formula for the trace of Hecke operators on spaces of modular forms for finite index subgroups of the modular group.

This is joint work with Don Zagier.

Elliptic curves and combinatorial objects

Mohammad Sadek

American University in Cairo, Egypt

Abstract

Let \( E \) be an elliptic curve defined over the rational field \( \mathbb{Q} \). We use the formal group associated to \( E \) in order to express the invariant differential of \( E \) as a power series. This power series is used to find many congruence relations satisfied by the coefficients of the \( L \)-series attached to \( E \). Surprisingly we moreover obtain many congruence relations satisfied by several binomial coefficients.

Arithmetic duality theorems: old and new

Tamas Szamuely

”Alfréd Rényi” Institute of Mathematics, Hungarian Academy of Sciences, Budapest

Abstract

Duality theorems for Galois and étale cohomology are among the most fundamental results in arithmetic. I shall recall some classic and some more recent ones, including a very
recent joint result with D. Harari concerning tori defined over the function field of a curve over a p-adic field.