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Project title: Homological and combinatorial aspects in the study of monomial ideals

Type: Postdoctoral research projects–PD

Scientific area: Sciences - Mathematics

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I. Project summary

Addressing a current topic research, the project aims to develop and elaborate new methods in the study of monomial ideals. Being at the intersection between commutative algebra and combinatorial algebra, monomial ideals can be studied by using techniques from both areas. There exist examples which show that homological properties and the invariants are not preserved by taking powers of ideals. Concerning the resolution, there are known necessary conditions for powers of monomial ideals with linear resolutions to have linear resolutions. One of the principal objectives of this project is to give conditions in order to characterize the linearity of the resolutions for powers of some ideals belonging to various classes of monomial ideals.

Combinatorial algebra is closely related to squarefree monomial ideals which are the algebraic point of view in the study of simplicial complexes. One of the most efficient methods of study is the Alexander duality. A particular class of simplicial complexes with applications in various areas is that of graphs. One may consider many classes of graphs, some of them arising from their combinatorial structure (trees, forests, bipartite graphs, chordal graphs) and other coming from the algebraic properties of their edge ideals (Cohen-Macaulay graphs, sequentially Cohen-Macaulay graphs). The research papers published in this area take into account the connections between these two types of classifications. An important concept in the graph theory is that of vertex cover of a graph. Vertex covers are important from both combinatorial and algebraic points of view since the minimal prime ideals of their edge ideals correspond to their minimal vertex covers. Extending the notion of vertex cover to simplicial complexes and hypergraphs leads to new connections with commutative algebra, especially with the powers of ideals, and, in this direction, goes another important objective of the project.

II. Published/accepted/submitted scientific results in 2010

[1] V. Ene, A. Olteanu, *Powers of lexsegment ideals with linear resolution*, to appear in Illinois Journal of Mathematics.

Abstract: All powers of lexsegment ideals with linear resolution (equivalently, with linear quotients) have linear quotients with respect to suitable orders of the minimal monomial generators. For a large subclass of lexsegment ideals the corresponding Rees algebra has a quadratic Gröbner basis, thus it is Koszul. We also find other classes of monomial ideals with linear quotients whose powers

have linear quotients too.

[2] A. Olteanu, *Normally torsion-free lexsegment ideals*, to appear in Algebra Colloquium.

Abstract: In this paper we characterize all the lexsegment ideals which are normally torsion-free. This will provide a large class of normally torsion-free monomial ideals which are not squarefree. Our characterization is given in terms of the ends of the lexsegment. We also prove that, for lexsegment ideals, the property of being normally torsion-free is equivalent to the property of the depth function of being constant.

III. Published/accepted/submitted scientific results in 2011

[1] A. Olteanu, *Monomial cut ideals*, to appear in Communications in Algebra.

Abstract: B. Sturmfels and S. Sullivant associated to any graph a toric ideal, called the cut ideal. We consider monomial cut ideals and we show that their algebraic properties such as the minimal primary decomposition, the property of having a linear resolution or being Cohen–Macaulay may be derived from the combinatorial structure of the graph.

[2] V. Micale, A. Olteanu, *On the Betti numbers of some semigroup rings*, Le Matematiche **67**(1), 2012, 145–159..

Abstract: For any numerical semigroup S, there are infinitely many numerical symmetric semigroups T such that $S = \frac{T}{2}$ is their half. We are studying the Betti numbers of the numerical semigroup ring $K[T]$ when $S = \frac{T}{2}$ is a 3-generated numerical semigroup or telescopic. We also consider 4-generated symmetric semigroups and the so called 4-irreducible numerical semigroups.

IV. Published/accepted/submitted scientific results in 2012

[1] A. Olteanu, *On the minimal graded free resolution of powers of lexsegment ideals*, arXiv 1206.6731, submitted.

Abstract: We consider powers of lexsegment ideals with a linear resolution (equivalently, with linear quotients) which are not completely lexsegment ideals. We give a complete description of their minimal graded free resolution.